College Invigilation Scheme towards Multi-Objective Scheduling

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Abstract: Today, the number of exams and students in colleges are quite large, while the number of in-service teachers is limited. Accordingly, the invigilation scheduling and arrangement becomes a difficult and complicated resource management problem. In existing scenarios, due to the consideration of various factors, this heavy work is done manually by the staffs and the result is always not very satisfactory as well. In this paper, we propose a multi-objective optimization scheme to balance the workload of each in-service teacher, to minimize the total workload, to provide sufficient rest time for teachers, and to make the best invigilation effect. Then, we use Lingo to obtain the locally optimal solution for the invigilation in Zhejiang University of Technology. The results validate its effectiveness and operability.

1. Introduction

The exams' invigilation arrangement is critical to resource management in the colleges [1-5]. We study the scenario of Chinese colleges' invigilation. A multi-objective optimization model is formulated for the in-service teachers and the exams to solve the reasonable invigilation arrangement problem. We suppose that two invigilators are required to be arranged in the examination room with fewer than 30 people. Three invigilators are required for the examination room with more than 70 people. For the examination room with 30-70 people, we arrange 2-3 invigilators. The examination time is divided into 24 time periods from the morning to the afternoon. In order to meet the criteria of using in-service teachers as much as possible, we stipulate that when the in-service teachers in a certain period of time are insufficient to complete the invigilation task, external teachers are invited to invigilate. The in-service teachers, exams, and 24 time periods are numbered respectively, and the data is pre-processed to obtain the number of exams per time period and the minimum number of invigilators required. Firstly, since one teacher is invigilated for one exam at the same time, with the goal of equalizing the teachers' workload and minimizing the total workload, the workload allocation model can be derived for 24 time periods to get the number of exams each teacher will invigilate. Based on it, in order to avoid the case that the teacher is overworked for continuous invigilation and lacks adequate rest time, we consider minimizing the variance of the teacher's invigilation date and obtain the teachers' specific identifier who participate the invigilation of each period. Finally, for each period, we maximize the invigilation effect, i.e., using the number of invigilated student per teacher is minimized and obtaining the invigilators' specific identifier for each invigilation, i.e., our invigilation management scheme.

2. Our invigilation management scheme

The whole optimization process is shown in Fig. 1.

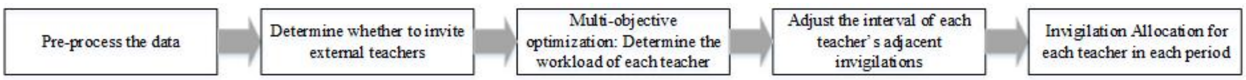

Figure 1. The procedure of our scheme

(1) Pre-process the data.

Due to the large number of exams, we classify the exams according to their exam dates. In most colleges, the duration of invigilation is 12 days and we define the morning and the afternoon of a day as a period, thus, the number of periods is 24. Let $S=\{s_1,s_2,\cdots,s_N\}$ where s_i denotes the number of students in the *i*-th exam and *N* denotes the total number of exams and $H=\{h_1,h_2,\cdots,h_N\}$ where h_i denotes the minimal number of invigilators in the *i*-th exam. According to our design, it holds that $h_i=\begin{cases}2, & s_i\le 70\\ 3, & s_i>70\end{cases}$. Let $T=\{t_1,t_2,\cdots,t_{24}\}$ and for each period *j*, we can obtain the minimal required number of invigilators t_j in the *j*-th period. Note that we also consider the case of two campuses. Let $B=\{b_1,b_2,\cdots,b_{24}\}$ and $C=\{c_1,c_2,\cdots,c_{24}\}$ denote the minimal required number of invigilators in the first campus and the second campus, respectively. Surely, it holds that $t_k=b_k+c_k$ (k=1,2,…,24).

Let $W=\{w_1,w_2,\cdots,w_{24}\}$ denote the number of invited external teachers for invigilation. When the number of invigilators required for a certain period is insufficient, in order to achieve the minimum number of external teachers, i.e.,

$$\min \max_{k=1,2,\cdots,24}\{w_k\} \tag{1}$$

where we adopt a greedy strategy, so that all in-service teachers are scheduled to participate in the invigilation, and we have

$$w_k=\begin{cases}t_k-M, & t_k>M\\ 0, & t_k\le M\end{cases} \tag{2}$$

$$\Leftrightarrow w_k=\max\{t_k-M,0\}$$

where *M* denotes the number of in-service teachers.

(2) Workload allocation model.

The workload of each teacher can be measured by the number of exams that the teacher invigilates. Since a teacher can only invigilate an exam at a time, the number of exams that the teacher invigilates is equal to the number of periods in which the teacher participates the invigilation task.

Let a_{kj} denote the 0-1 variable to determine whether *j*-th teacher participates in the *k*-th period's invigilation task, p_{kj} denote 0-1 variable determine whether *j*-th teacher participates in the *k*-th period's invigilation task in the first campus, z_{kj} denote 0-1 variable determine whether *j*-th teacher participates in the *k*-th period's invigilation task in the second campus. It satisfies that $a_{kj}=p_{kj}+z_{kj}$ (k=1, 2,…, 24).

The number of invigilators at each time period should meet at least the minimum requirements. For the period in which the minimum required number of invigilators is less than *M*, the in-service teachers are scheduled for invigilation. For the time period in which the number of invigilators required exceeds *M*, the in-service teachers are all involved in the invigilation, and then the external teachers are invited to fill the vacant invigilation positions. Therefore, the constraints are as follows:

$$\sum_{j=1}^{M} p_{kj}\ge b_k \text{ and } \sum_{j=1}^{M} z_{kj}\ge c_k \text{, if } t_k\le M \tag{3}$$

$$\text{For } \forall j\in[1,M],\ a_{kj}=1\text{, if } t_k>M \tag{4}$$

In order to facilitate the arrangement of academic affairs and the personal convenience of teachers, we make the in-service teachers in the afternoon invigilation as far as possible from the in-service teachers who participated in the invigilation in the morning. The exam date is numbered from 1 to 12 in chronological order and we define v=1, 2,…, 12 to denote it. Then we have

$$\text{For } \forall v, \ \sum_{j=1}^{M} p_{2v-1,j} p_{2v,j} \geq \min\{b_{2v-1}, b_{2v}\} \ \text{ and } \ \sum_{j=1}^{M} z_{2v-1,j} z_{2v,j} \geq \min\{c_{2v-1}, c_{2v}\} \tag{5}$$

Since trans-campus invigilation in the morning and afternoon will cause great inconvenience and fatigue for the in-service teachers, we arrange for the teachers to invigilate in the same campus within one day. So

$$\text{For } v = 1, 2, \cdots, 12, \ \forall j \in [1, M], \ p_{2v-1,j} z_{2v,j} = 0 \ \text{ and } \ z_{2v-1,j} p_{2v,j} = 0 \tag{6}$$

We assume that one teacher to participate in a period of invigilation is the unit workload, and the total workload of one teacher is $\sum_{k=1}^{24} a_{kj}$, j=1,2,…,M.

On the one hand, the workload of teachers should be balanced. On the other hand, the total workload of teachers should be minimized. Thus we have the following workload allocation optimization model:

$$\min \ \alpha\left(\max_{1\leq j\leq M}\left\{\sum_{k=1}^{24} a_{kj}\right\} - \min_{1\leq j\leq M}\left\{\sum_{k=1}^{24} a_{kj}\right\}\right) + \beta \frac{\sum_{j=1}^{M}\sum_{k=1}^{24} a_{kj}}{M}$$

$$s.t.\begin{cases} \sum_{j=1}^{M} p_{kj} \geq b_k, \sum_{j=1}^{M} z_{kj} \geq c_k, t_k \leq M \\ \forall j \in [1, M], j \in N^*, t_k > M, a_{kj} = 1 \\ h_i = \begin{cases} 2, & s_i \leq 70 \\ 3, & s_i > 70 \end{cases} \\ \forall k \in [1, 24], k \in N^*, j = 1, ..., M, a_{kj} = p_{kj} + z_{kj} \\ \forall v \in [1,12], v \in N^*, j = 1, ..., M, \sum_{j=1}^{M} p_{2v-1,j} p_{2v,j} \geq \min\{b_{2v-1}, b_{2v}\}, t_k \leq M \\ \forall v \in [1,12], v \in N^*, j = 1, ..., M, \sum_{j=1}^{M} z_{2v-1,j} z_{2v,j} \geq \min\{c_{2v-1}, c_{2v}\}, t_k \leq M \\ \forall j \in [1, M], j \in N^*, v = 1, 2, \cdots 12, p_{2v-1,j} z_{2v,j} = 0, t_k \leq M \\ \forall j \in [1, M], j \in N^*, v = 1, 2, \cdots 12, z_{2v-1,j} p_{2v,j} = 0, t_k \leq M \end{cases}$$

where α and β is the weights of two optimization objectives.

(3) Invigilation Date Determination model.

It is obviously unreasonable for a teacher to keep invigilating for several days. According to the analysis, the more dispersed the teacher’s invigilation date is, the easier the teacher's work is, that is, the variance of the date label of each teacher’s invigilation should be as large as possible. After determining the allocation of the workload, on the basis of not changing the balance indicator and the total workload of the teacher, the invigilation date of a teacher is dispersed as much as possible by adjusting the invigilation dates between the two teachers. For example, a teacher is working on three dates numbered 2, 5, and 7 and thus is easier than working on three dates numbered 2, 3, and 4. It can be seen that the larger the variance of the invigilation date label is, the more dispersed the invigilation date is. Therefore, we aim to maximize the minimum value of the variance of the invigilation date label, and the objective function is as follows:

$$\max \min_{1\le j\le M} \sum_{v=1}^{12}\left(\left(a_{2v-1,j}+a_{2v,j}-a_{2v-1,j}a_{2v,j}\right)v-\frac{\sum_{v=1}^{12}\left(a_{2v-1,j}+a_{2v,j}-a_{2v-1,j}a_{2v,j}\right)v}{\sum_{v=1}^{12}\left(a_{2v-1,j}+a_{2v,j}-a_{2v-1,j}a_{2v,j}\right)}\right)^2 \tag{7}$$

where $\frac{\sum_{v=1}^{12}\left(a_{2v-1,j}+a_{2v,j}-a_{2v-1,j}a_{2v,j}\right)v}{\sum_{v=1}^{12}\left(a_{2v-1,j}+a_{2v,j}-a_{2v-1,j}a_{2v,j}\right)}$ means the average value of *j*-th teacher's invigilation date labels.

From above workload allocation model, we can obtain the workload of each teacher e_j, i.e.,

$$\sum_{k=1}^{24} a_{kj} = e_j \text{ , } j\text{=}1,2,\ldots,M \tag{8}$$

Combined with the constraints (5) and (6), the invigilation date determination model is given as follows:

$$\max \min_{1\le j\le M} \sum_{v=1}^{12}\left(\left(a_{2v-1,j}+a_{2v,j}-a_{2v-1,j}a_{2v,j}\right)v-\frac{\sum_{v=1}^{12}\left(a_{2v-1,j}+a_{2v,j}-a_{2v-1,j}a_{2v,j}\right)v}{\sum_{v=1}^{12}\left(a_{2v-1,j}+a_{2v,j}-a_{2v-1,j}a_{2v,j}\right)}\right)^2$$

$$s.t.\begin{cases}\sum_{k=1}^{24} a_{kj} = e_j,\ j=1,...,M \\ \forall k\in[1,24], k\in N^*,\ j=1,...,M,\ a_{kj} = p_{kj}+z_{kj} \\ \forall v\in[1,12], v\in N^*,\ j=1,...,M,\ \sum_{j=1}^{M} p_{2v-1,j}p_{2v,j} \ge \min\left\{b_{2v-1}, b_{2v}\right\}, t_k \le M \\ \forall v\in[1,12], v\in N^*,\ j=1,...,M,\ \sum_{j=1}^{M} \mathrm{z}_{2v-1,j}\mathrm{z}_{2v,j} \ge \min\left\{c_{2v-1}, c_{2v}\right\}, t_k \le M \\ \forall j\in[1,M], j\in N^*, v=1,2,\cdots 12,\ p_{2v-1,j}z_{2v,j}=0, t_k\le M \\ \forall j\in[1,M], j\in N^*, v=1,2,\cdots 12,\ z_{2v-1,j}p_{2v,j}=0, t_k\le M \end{cases}$$

(4) Specific Exams Allocation model.

After determining the invigilation dates of each teacher, we consider to allocate the specific exam to each teacher.

For an examination room, the effect of the invigilation should be as good as possible, that is, the average number of students over the number of invigilators in an exam should not be too large.

Let x_{ij} denote 0-1 variable determining whether *j*-th teacher will invigilate *i*-th exam. We number each exam in chronological order and denote $Q=\left\{q_1,q_2,\cdots,q_{25}\right\}$ where $q_k\left(k\text{=}1,2,\cdots,24\right)$ means the first exam in the *k*-th period and we set $q_{25}=N+1$. Denote the number of invited external teachers for each exam by $L=\{l_i,l_2,\cdots,l_N\}$. For *i*-th exam, the number of students per teacher invigilates is $\frac{s_i}{\sum_{j=1}^{M} x_{ij} + l_i}$. Denote $O = \{o_1, o_2, \cdots, o_N\}$ and o_i is 0-1 variable to illustrate whether *i*-th exam is in the first campus. Then, combined with the obtained results of invigilation date determination we have following constraints:

$$\sum_{i=q_k}^{(q_{k+1}-1)} x_{ij} = \begin{cases} p_{kj}, & o_i = 1 \\ z_{kj}, & o_i = 0 \end{cases}, \quad j = 1, 2, \cdots, M \tag{9}$$

For the k-th period, for each exam, the sum of the number of in-service teachers participating and the number of external teachers participating should be no less than the minimum required number of invigilators, i.e.,

$$l_i + \sum_{j=1}^{M} x_{ij} \geq h_i, \quad i = q_k, q_k + 1, \cdots, q_{k+1} - 1 \tag{10}$$

Meanwhile, for the *k*-th period, the total number of invited external teachers should be equal to the one obtained in the data pre-processing, i.e.,

$$\sum_{i=q_k}^{(q_{k+1}-1)} l_i = w_k, k = 1, 2, ..., 24 \tag{11}$$

In summary, we present our specific exams allocation model of each period as follows:

$$\min \quad \max_{q_k \leq i \leq (q_{k+1}-1)} \left\{ \frac{s_i}{\sum_{j=1}^{M} x_{ij} + l_i} \right\}$$

$$s.t. \begin{cases} \sum_{i=q_k}^{(q_{k+1}-1)} x_{ij} = \begin{cases} p_{kj}, & o_i = 1 \\ z_{kj}, & o_i = 0 \end{cases}, \quad j = 1, 2, \cdots, M \\ l_i + \sum_{j=1}^{M} x_{ij} \geq h_i, \quad i = q_k, q_k + 1, \cdots, q_{k+1} - 1 \\ \sum_{i=q_k}^{(q_{k+1}-1)} l_i = w_k \end{cases}$$

3. Test and Evaluation of Our Scheme

We implement our scheme on the final exams of mathematics department in the Zhejiang University of Technology, which contains 80 teachers required to be allocated and 225 exams [6].

We use Lingo to solve above three optimization models and obtain the locally optimal solution of our proposed scheme. For example, the result of workload allocation is shown in Table 1, which outperforms the random workload allocation scheme.

Table. 1. Result of workload allocation

Workload	Number of teachers	The identifier of teachers
4	17	36 52 53 54 55 56 57 59 60 62 63 65 66 67 69 74 78
5	37	3 6 7 8 9 11 12 13 23 24 26 27 28 31 32 33 35 37 40 41 42 44 45 46 47 50 58 61 64 68 71 73 75 76 77 79 80
6	26	1 2 4 5 10 14 15 16 17 18 19 20 21 22 25 29 30 34 38 39 43 48 49 51 70 72

4. Conclusion

We study the problem of the exams' invigilation arrangement and propose a multi-objective optimization scheme to balance the workload of each in-service teacher, to minimize the total workload, to provide sufficient rest time for teachers, and to make the best invigilation effect.

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